

# Mitigating Data Sparsity Using Similarity Reinforcement-Enhanced Collaborative Filtering

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The data sparsity problem has attracted significant attention in collaborative filtering-based recommender systems. To alleviate data sparsity, several previous efforts employed hybrid approaches that incorporate auxiliary data sources into recommendation techniques, like content, context, or social relationships. However, due to privacy and security concerns, it is generally difficult to collect such auxiliary information. In this article, we focus on the pure collaborative filtering methods without relying on any auxiliary data source. We propose an improved memory-based collaborative filtering approach enhanced by a novel similarity reinforcement mechanism. It can discover potential similarity relationships between users or items by making better use of known but limited user-item interactions, thus to extract plentiful historical rating information from similar neighbors to make more reliable and accurate rating predictions. This approach integrates user similarity reinforcement and item similarity reinforcement into a comprehensive framework and lets them enhance each other. Comprehensive experiments conducted on several public datasets demonstrate that, in the face of data sparsity, our approach achieves a significant improvement in prediction accuracy when compared with the state-of-the-art memory-based and model-based collaborative filtering algorithms.

CCS Concepts: • **Information systems → Collaborative filtering; Nearest-neighbor search; Social recommendation;**

Additional Key Words and Phrases: Recommender system, rating prediction, personalization, data sparsity, similarity reinforcement

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## 1. INTRODUCTION

Modern customers are faced with a variety of choices. Content providers and online retailers provide a huge selection of information and products, with unprecedented

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opportunities to satisfy all kinds of tastes and needs. Recommender systems [36, 37] provide an excellent solution for solving the problem of information overloading. They aim to study the relationships between users and items, reveal users' preferences on items, and recommend the most suitable items to users. Commercial applications of recommender systems include e-commerce (e.g., Amazon, eBay) [40], fashion [13, 44], food and restaurants [34], social events [31], and art (e.g., movies, music, books) [5, 26, 45]. In these areas, recommender systems providing excellent personalized suggestions significantly increase the likelihood that a consumer purchases a product or selects an item compared with unpersonalized suggestions.

Collaborative Filtering (CF) [15] is one of the most commonly used technologies in recommender systems. The intuition behind CF is that similar users share similar interests and preferences (user-based) and similar items have similar characteristics (item-based). User-based methods find similar neighbors for an active user by studying the similarities between users' preferences. After that, recommender systems predict the unknown preferences of the active user on a set of candidate items and generate a ranking list of items, that the active user will like the most. Item-based methods reveal relationships between items and use historical information from similar items to make preference predictions. The biggest advantage of CF is that it does not rely on the content and form of items and can be applied to items without textual description.

Generally speaking, there are two types of CF algorithms: memory-based [8, 38, 48, 49, 57] and model-based [2, 6, 17, 43]. Memory-based algorithms construct a user-item rating matrix based on collected rating data. Each element of the matrix represents a user's rating on an item. The algorithms attempt to discover relationships between users or items by calculating similarities between users or items. After that, they predict the unknown preference of an active user based on the past information provided by similar users and items. Model-based algorithms either explore a "latent space" or build a model to capture the user-item relationships. They use existing data to train the pre-defined model and make preference predictions based on the trained model. Regression [46], matrix factorization [28], and Bayesian methods [16] are popular approaches that fall into this category. They have a good theoretical basis, but are not so intuitive in terms of interpreting prediction results as memory-based algorithms are [49]. This article focuses on memory-based CF algorithms.

It has been demonstrated that memory-based CF algorithms have achieved considerable success in recommender systems. However, they suffer from the data sparsity problem [10]. In real-world applications, most users only rate a small number of items and most items are only rated by a small number of users, so the rating matrix is quite sparse. In this case, it has been proved that traditional similarity measures, such as Pearson Correlation Coefficient (PCC) and Cosine-based similarity, cannot function well [29]. Therefore, the data sparsity problem leads to great difficulties in finding similar neighbors, and prevents memory-based CF algorithms from making accurate preference predictions. This problem was partially solved by incorporating auxiliary data sources, which are relatively dense, such as the content of items [50], users' locations [53], and users' friendship or social trust information [14, 18–20, 30]. However, the introduction of auxiliary information complicates system implementation. Additionally, due to privacy and security concerns, users may not be willing to provide their personal information, such as their locations or social relationships. In this article, we focus on a pure memory-based CF approach and propose a novel similarity reinforcement mechanism to mitigate the data sparsity problem. Our approach does not require any auxiliary information. It only relies on known user-item interactions and yet achieves significant improvement on prediction accuracy compared with the state-of-the-art CF algorithms. The key contributions of this work are summarized as follows:

- A novel similarity reinforcement mechanism is proposed to enhance the memory-based CF algorithm so that more potential similarity relationships can be discovered between users or items, and more historical information can be extracted from similar neighbors to lessen data sparsity.
- A comprehensive algorithm is designed to integrate user similarity reinforcement and item similarity reinforcement and let the two procedures enhance each other.
- The effectiveness of the proposed approach to handle data sparsity has been validated by the experiments conducted on three publicly available datasets: MovieLens 1M<sup>1</sup>, Netflix<sup>2</sup>, and Yahoo! Music<sup>3</sup>.

The rest of the article is organized as follows. Section 2 introduces some related work. In Section 3, we present the basic notations used throughout the article and the formal problem definition. In Section 4, a memory-based CF approach enhanced by the novel similarity reinforcement mechanism is proposed to alleviate the data sparsity problem in recommender systems. Section 5 conducts comprehensive experiments to validate the effectiveness of our approach. Section 6 discusses the computational complexity of the proposed approach. Finally, conclusions are drawn in Section 7.

## 2. RELATED WORK

CF technology has been developed since the early 1990s. It was employed in the Tapestry system [9] to filter mails from several mailing lists by analyzing the reactions (annotations) given by collaborative users to documents they read. Since then, many CF algorithms have been proposed. They are generally categorized into two classes: memory-based and model-based. Memory-based approaches utilize historical user-item rating data to estimate the similarities between users or items. Based on that, historical rating information from similar neighbors are collected to predict the preferences of an active user on a set of candidate items. The most popular memory-based CF approaches include user-based [3, 21], item-based [7, 38], and hybrid [56, 57] algorithms. User-based algorithms use the historical rating data from similar users for preference prediction, while item-based methods extract data from similar items for prediction. The hybrid CF approach integrates the two former methods, and achieves a better prediction performance. Model-based CF algorithms construct appropriate models (e.g., matrix factorization models [25, 41], clustering models [43], and latent factor models [4]) for user-item relationships. Model training is generally performed offline to determine the optimal values of model parameters. Finally, the trained models are used for preference prediction.

In recent years, based on traditional CF approaches, some new methods like feature engineering and multi-layered neural networks were proposed for recommender systems. Rendle et al. [35] argued that applying factorization approaches to a new prediction problem is a nontrivial task and requires a lot of expert knowledge. Therefore, they proposed factorization machines (FM), a generic and easy-to-use approach. Factorization machines can mimic most factorization models by feature engineering and combine the generality and flexibility of feature engineering with the high prediction accuracy of factorization models. Wu et al. [51] used a three-layer neural network to reconstruct the full user-item preferences by learning the latent representations of the partially observed user-item rating data. They also proved that the proposed model is a generalization of several well-known collaborative filtering models, like latent

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<sup>1</sup><http://grouplens.org>.

<sup>2</sup><http://www.netflixprize.com>.

<sup>3</sup><http://webscope.sandbox.yahoo.com/#datasets>.

factor model, similarity model, and factorized similarity model, but with more flexible components.

Although CF has achieved significant success in recommender systems, it still has the problems of scalability and data sparsity when applied in the real world. Wu et al. [52] tried to deal with the problem of scalability by clustering users and items into several subgroups. Each subgroup includes a set of like-minded users and their liked items. Traditional CF methods are applied to each subgroup, and the recommendation results from all subgroups can be easily aggregated. Zhang et al. [55] presented a Localized Matrix Factorization (LMF) framework. They attempted to overcome the challenges of scalability and sparsity by transforming a large sparse matrix into Recursive Bordered Block Diagonal Form (RBBDF) and extracting smaller and dense submatrices from RBBDF. This approach has the potential to improve prediction accuracy by factorizing these submatrices independently. This framework is also suitable for parallelization and thus improves system scalability. Jing et al. [22] proposed a sparse probabilistic matrix factorization (SPMF) method by using the Laplacian distribution to model the user/item factor vector. The Laplacian distribution has the ability to generate sparse coding. Additionally, the heavy tails in the Laplacian distribution help to identify the tail items and partially solve the long tail problem.

Other studies have used auxiliary information to deal with the data sparsity problem, including item semantics and content information, or users' social relationships (e.g., friendship or trust). Moshfeghi et al. [32] imported emotion and semantic features related to items into recommender systems. They extracted two emotion spaces from movie plot summaries and review data, and considered three semantic spaces: actor, director, and genre. Additionally, they employed a framework which relies on an extended Latent Dirichlet Allocation (LDA) model for prediction. They concluded that actor and movie spaces are the least and most sensitive spaces to dataset size and data sparsity, and the model that incorporates all spaces performs the best at remedying data sparsity compared with state-of-the-art CF systems. Wang et al. [47] proposed a hierarchical Bayesian model called Collaborative Deep Learning (CDL), which jointly performs deep representation learning for item content and collaborative filtering for the rating matrix. By performing deep learning collaboratively, CDL can simultaneously extract an effective deep feature representation from content and capture similarities and implicit relationships between items or users. Pan et al. [33] attempted to handle the issue of data sparsity in a target domain by transferring knowledge about both users and items from other related auxiliary data sources that are relatively dense. They used a matrix-based transfer learning technology, which is capable of dealing with the data heterogeneity in different domains.

Kaya and Alpaslan [23] extracted like-minded ideas from social networks to mitigate the problem of data sparsity. Furthermore, their study verified that social networks correlated with a specific domain carry better information and knowledge required to make recommendations in that specific domain. Konstas et al. [24] considered for music track recommendation both the social annotations (tags) and friendship relations in a social graph constructed among users, items, and tags. Because of the vagueness of friendship, friends may show different opinions on items. However, users participating in the same actual or virtual communities are more inclined to share similar interests [54]. Therefore, the data extracted from both friendship and membership are leveraged and fused for more accurate preference prediction.

Compared with membership and friendship, trust information is less vague and can better facilitate the discovery of similarity relationships. In research literature, trust information has been adopted in many recommendation approaches. Haydar et al. [14] combined opinion similarity and trust similarity to increase the connectivity between users, and reduced the impact of data sparsity. Guo et al. [11, 12] utilized the merged

	$i_1$	$i_2$	$i_3$	$\dots$	$i_N$
$u_1$	$r_{11}$	?	$r_{13}$	$\dots$	?
$u_2$	$r_{21}$	?	?	$\dots$	$r_{2N}$
$u_3$	?	$r_{32}$	?	$\dots$	?
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$u_M$	?	$r_{M2}$	?	$\dots$	$r_{MN}$

Fig. 1. Rating matrix.

ratings from trusted neighbors to compensate for the incomplete preference of an active user. Jamali and Ester [18] designed a random walk mechanism to combine the trust-based and item-based CF approaches for recommendation. Additionally, they applied the trust propagation technique to the matrix factorization models in a subsequent study [19]. Ma et al. [30] designed a novel probabilistic factor analysis model, in which user favors and the preferences of their trusted friends are well merged together.

One drawback of the approaches based on auxiliary sources is the high dependency on auxiliary information. It increases the storage cost and complicates the implementation of recommender systems. Furthermore, due to privacy and security concerns, it is generally difficult to collect users' trust, friendship, or membership information. Hence, it is important to design effective algorithms only based on the rating information for higher-quality recommendation. Liu et al. [27] focused on memory-based CF and proposed an improved heuristic similarity measure only based on users' rating data. The model provides a more accurate similarity measure by taking into account several factors of similarity, namely Proximity, Impact, Popularity, and users' rating preferences. However, it still relies on the co-rated items between users. If two users have no items rated in common, their similarities still cannot be estimated. In this article, we focus on a pure memory-based CF algorithm without using any auxiliary data source. We designed a novel similarity reinforcement mechanism, that can reveal implicit similarities between users or items. Compared with traditional CF methods, our method improves the prediction accuracy significantly and alleviates the data sparsity problem effectively.

### 3. NOTATION AND PROBLEM DEFINITION

Before presenting the proposed approach, we first introduce the basic notations and concepts used throughout the article and then formally describe the problem to be solved in recommender systems.

We use  $[N]$  to denote a set of consecutive positive integers  $\{1, 2, \dots, N\}$ . Scalars are denoted by lowercase letters such as  $a$ . Vectors are denoted by boldface lowercase letters such as  $\mathbf{u}$ . Moreover, we use boldface uppercase letters such as  $\mathbf{M}$  to denote matrices. The Frobenius norm of a matrix  $\mathbf{M} \in \mathbb{R}^{m \times n}$  is denoted by  $\|\mathbf{M}\|_{\text{F}}$ , i.e.,  $\|\mathbf{M}\|_{\text{F}} = \sqrt{\sum_{i=1}^m \sum_{j=1}^n |\mathbf{M}_{ij}|^2}$ .

In a recommender system, there is a set of  $M$  users  $\mathcal{U} = \{u_1, u_2, \dots, u_M\}$  and a set of  $N$  items  $\mathcal{I} = \{i_1, i_2, \dots, i_N\}$ . The interactions between users and items are summarized in a  $M \times N$  rating matrix  $\mathbf{R}$ , as depicted in Figure 1. Each row of the matrix corresponds

to a user and each column corresponds to an item. The  $(p, q)$  entry  $r_{pq}$  ( $p \in [M]$ ,  $q \in [N]$ ) in the matrix denotes the rating value given by user  $u_p$  to item  $i_q$ . If a user did not rate an item, the corresponding entry is called a missing value.

Traditional memory-based CF algorithms utilize the observed values in the rating matrix to predict missing values. When a missing rating value is predicted for user  $u_p$  on item  $i_q$ ,  $u_p$  and  $i_q$  are called an active user and a candidate item, respectively. A typical memory-based CF recommendation problem involves three main steps. In the first step, the similarities between users or items are estimated using some similarity measures, such as PCC and Cosine-based similarities. In the second step, the algorithm finds a set of similar users for the active user and a set of similar items for the candidate item, where similar users/items are those with positive similarities. In the third step, the preferences of similar users on the candidate item and the preferences of the active user on similar items are combined to predict the missing rating value for the active user on the candidate item. In the rest of the article, we interchangeably use the terms “similar users/items” and “users/items with positive similarities.”

However, we note that the rating matrix is only partially filled with observed values. The percentage of missing values in a matrix is called the *sparsity level* of the matrix. The sparsity level of the user-item rating matrix in real-world recommender systems usually exceeds 90%. This brings great challenges to similarity calculation and rating prediction within memory-based CF algorithms. In the following section, we propose a novel similarity reinforcement approach to reduce the impact of data sparsity on recommender systems.

#### 4. SIMILARITY REINFORCEMENT-ENHANCED COLLABORATIVE FILTERING

In this section, we first explain the underlying motivations and assumptions of our approach and then present the proposed approach, named Similarity Reinforcement-Enhanced Collaborative Filtering (SRCF).

##### 4.1. Similarity Computation

As mentioned previously, the similarity computing between users or items is a core issue in memory-based CF algorithms. Well-known similarity measures include PCC, constrained PCC, Cosine-based similarity, adjusted Cosine-based similarity, and other improved versions [1]. To introduce our approach, we use the PCC similarity as an example and our techniques can be applied to other similarity measures in a similar way. In user-based CF methods, the PCC similarity between two users  $u_a$  and  $u_b$  is calculated by the following equation:

$$\text{sim}'(u_a, u_b) = \frac{\sum_{i_k \in \mathcal{I}_{ab}} (r_{ak} - \bar{r}_a)(r_{bk} - \bar{r}_b)}{\sqrt{\sum_{i_k \in \mathcal{I}_{ab}} (r_{ak} - \bar{r}_a)^2} \sqrt{\sum_{i_k \in \mathcal{I}_{ab}} (r_{bk} - \bar{r}_b)^2}}, \quad (1)$$

where  $\mathcal{I}_{ab} = \mathcal{I}_a \cap \mathcal{I}_b$  is the subset of items rated by both  $u_a$  and  $u_b$ , and  $\bar{r}_a$  and  $\bar{r}_b$  are the average ratings given by  $u_a$  and  $u_b$ , respectively. With this definition, the similarity between two users is in the interval  $[-1, 1]$ , with a larger PCC value indicating a higher similarity value. Likewise, in item-based CF methods, the PCC similarity between two items  $i_k$  and  $i_l$  is computed by:

$$\text{sim}'(i_k, i_l) = \frac{\sum_{u_a \in \mathcal{U}_{kl}} (r_{ak} - \bar{r}_k)(r_{al} - \bar{r}_l)}{\sqrt{\sum_{u_a \in \mathcal{U}_{kl}} (r_{ak} - \bar{r}_k)^2} \sqrt{\sum_{u_a \in \mathcal{U}_{kl}} (r_{al} - \bar{r}_l)^2}}, \quad (2)$$

where  $\mathcal{U}_{kl} = \mathcal{U}_k \cap \mathcal{U}_l$  is the subset of users who rated both  $i_k$  and  $i_l$ , and  $\bar{r}_k$  and  $\bar{r}_l$  are the average ratings of items  $i_k$  and  $i_l$ , respectively.

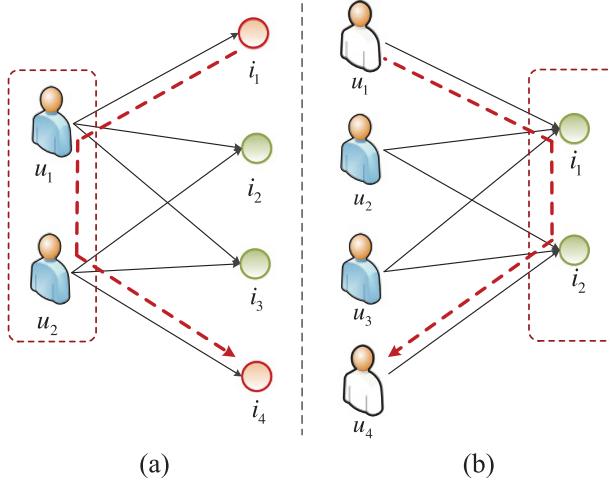


Fig. 2. An example for similarity reinforcement.

Equation (1) often overestimates the similarities between two users who are not similar actually but happen to give similar ratings to a small number of co-rated items. To deal with this problem, a similarity weight [56, 57] is utilized to reduce the impact of a small number of co-rated items:

$$\text{sim}(u_a, u_b) = \frac{2|\mathcal{I}_{ab}|}{|\mathcal{I}_a| + |\mathcal{I}_b|} \cdot \text{sim}'(u_a, u_b), \quad (3)$$

where  $|\mathcal{I}_{ab}| = |\mathcal{I}_a \cap \mathcal{I}_b|$  is the number of items that  $u_a$  and  $u_b$  both rated,  $|\mathcal{I}_a|$  and  $|\mathcal{I}_b|$  are the numbers of items rated by  $u_a$  and  $u_b$ , respectively. Similarly, item similarity is also revised to reduce the impact of a small number of common users:

$$\text{sim}(i_k, i_l) = \frac{2|\mathcal{U}_{kl}|}{|\mathcal{U}_k| + |\mathcal{U}_l|} \cdot \text{sim}'(i_k, i_l), \quad (4)$$

where  $|\mathcal{U}_{kl}| = |\mathcal{U}_k \cap \mathcal{U}_l|$  is the number of users who rated both  $i_k$  and  $i_l$ ,  $|\mathcal{U}_k|$  and  $|\mathcal{U}_l|$  are the numbers of users who rated  $i_k$  and  $i_l$ , respectively.

## 4.2. Similarity Reinforcement

**4.2.1. Motivations.** As indicated by Equations (1) to (4), if two users/items have no item/user intersection, namely  $\mathcal{I}_{ab} = \text{null}$  or  $\mathcal{U}_{kl} = \text{null}$ , their similarity cannot be calculated. Unfortunately, due to the huge number of users and items in a recommender system, even very active users may rate only a few items and even very popular items may be rated by only a few users, so the rating matrix  $\mathbf{R}$  is quite sparse. Therefore, the traditional similarity measures cannot identify enough similar neighbors for active users or candidate items, which dramatically degrades the performance of CF algorithms. To address the problem, we designed a novel similarity reinforcement mechanism to compensate for the deficiency of traditional memory-based CF approaches. Our method aims to reveal implicit similarity relationships between users or items without any auxiliary information. Implicit similarity relationships can help to get more historical data from similar neighbors and achieve higher prediction accuracy.

The similarity reinforcement mechanism is mainly based on PCC similarities and observed user-item ratings. A toy example is shown in Figure 2. In Figure 2(a), there are two users,  $u_1$  and  $u_2$ , and four items  $i_1$  to  $i_4$ .  $u_1$  and  $u_2$  both rated  $i_2$  and  $i_3$ , while  $i_1$  was only rated by  $u_1$  and  $i_4$  was only rated by  $u_2$ . According to traditional similarity

	$i_1$	$i_2$	$i_3$	$i_4$	$i_5$
$u_1$	5	5	5	--	--
$u_2$	5	5	--	5	1

	$i_1$	$i_2$	$i_3$	$i_4$	$i_5$
$u_1$	5	1	5	--	--
$u_2$	1	5	--	5	1

(a)

(b)

Fig. 3. Similarity propagation.

measures, the similarity between  $i_1$  and  $i_4$  cannot be calculated, since the two items are not rated by any common user. However, we observe that  $i_1$  is rated by  $u_1$ , and  $i_4$  is rated by  $u_2$ , and  $u_1$  and  $u_2$  have a PCC similarity since they have two items  $i_2$  and  $i_3$  rated in common. Therefore,  $i_1$  and  $i_4$  can establish an implicit similarity relationship through the affinity between  $u_1$  and  $u_2$ . Likewise, as shown in Figure 2(b),  $u_1$  and  $u_4$ , who have no item rated in common, can build an implicit similarity relationship through the affinity between  $i_1$  and  $i_2$ . Therefore, our similarity reinforcement mechanism is premised on the following three assumptions:

- Users with no direct PCC similarities can build implicit similarities through the affinity between their items.
- Items with no direct PCC similarities can build implicit similarities through the affinity between their users.
- User similarity reinforcement and item similarity reinforcement can enhance each other.

The next problem is how similarities propagate between users or items without direct PCC similarities. We use two examples in Figure 3 to illustrate our basic idea. We assume that users' ratings range from 1 to 5, where 1 means "dislike very much" and 5 means "like very much." In Figure 3(a),  $u_1$  and  $u_2$  give a similar rating of 5 to their co-rated items  $i_1$  and  $i_2$ . As a result,  $u_1$  and  $u_2$  have a positive similarity, as do  $i_1$  and  $i_2$ . On the other hand,  $u_1$  rates  $i_3$  as 5 and  $u_2$  rates  $i_4$  as 5. Although  $i_3$  and  $i_4$  have no common user, it is reasonable to establish an implicit positive similarity between  $i_3$  and  $i_4$ , since they receive similar ratings from two similar users, respectively. As for  $i_3$  and  $i_5$ , they receive totally different ratings from  $u_1$  and  $u_2$ , respectively, so a negative similarity should be built between  $i_3$  and  $i_5$ . In Figure 3(b),  $u_1$  rates  $i_1$  and  $i_2$  as 1 and 5, while  $u_2$  rates  $i_1$  and  $i_2$  as 5 and 1. As a result,  $u_1$  and  $u_2$  have a negative PCC similarity, as do  $i_1$  and  $i_2$ . As for the item pair  $i_3$  and  $i_4$ , they have no common user, and  $u_1$  rates  $i_3$  as 5 and  $u_2$  rates  $i_4$  as 5. We can infer that  $i_3$  is similar to  $i_1$  because the same user  $u_1$  rates both of them as 5, and  $i_4$  is similar to  $i_2$  because the same user  $u_2$  rates both of them as 5. However,  $i_1$  is dissimilar to  $i_2$ , so we can infer that  $i_3$  and  $i_4$  are also dissimilar. As for  $i_3$  and  $i_5$ ,  $i_5$  receives a rating of 1 from  $u_2$ , so  $i_5$  is similar to  $i_1$  since they both receive a rating of 1 from the same user  $u_2$ , and we can further infer that  $i_5$  is similar to  $i_3$  because they are both similar to  $i_1$ .

Based on the above analysis, we summarize four ways of similarity propagation between items without user intersection: (i) if two users with a positive similarity give similar ratings to two items, the items have a positive similarity; (ii) if two users with a positive similarity give dissimilar ratings to two items, the items have a negative similarity; (iii) if two users with a negative similarity give similar ratings to two items, the items have a negative similarity; (iv) if two users with a negative similarity give dissimilar ratings to two items, the items have a positive similarity. The similarity propagation mechanism between users without item intersection is analogous.

**4.2.2. Similarity Reinforcement Algorithm.** Suppose  $u_a$  and  $u_b$  are two users. The item set rated by  $u_a$  is denoted by  $\mathcal{I}_a = \{i_1^a, i_2^a, \dots, i_P^a\}$  ( $\mathcal{I}_a \subset \mathcal{I}$ ), and the item set rated by  $u_b$  is denoted by  $\mathcal{I}_b = \{i_1^b, i_2^b, \dots, i_Q^b\}$  ( $\mathcal{I}_b \subset \mathcal{I}$ ). We define two sets  $\mathcal{PI}_{ab} = \{(i_p^a, i_q^b) \mid (i_p^a, i_q^b) \in \mathcal{I}_a \times \mathcal{I}_b, sim(i_p^a, i_q^b) > 0\}$ , which includes the item pairs from  $\mathcal{I}_a \times \mathcal{I}_b$  with positive similarities, and  $\mathcal{NI}_{ab} = \{(i_p^a, i_q^b) \mid (i_p^a, i_q^b) \in \mathcal{I}_a \times \mathcal{I}_b, sim(i_p^a, i_q^b) < 0\}$ , which consists of the item pairs from  $\mathcal{I}_a \times \mathcal{I}_b$  with negative similarities. Accordingly, the similarity reinforcement between  $u_a$  and  $u_b$  can be achieved based on the affinity between each pair of items  $(i_p^a, i_q^b)$  from  $\mathcal{PI}_{ab}$  or  $\mathcal{NI}_{ab}$ :

$$sim(u_a, u_b) = (1 - \alpha) \cdot sim(u_a, u_b) + \alpha \cdot \frac{\sum_{(i_p^a, i_q^b) \in \mathcal{PI}_{ab} \cup \mathcal{NI}_{ab}} w_{pq} \cdot sim(i_p^a, i_q^b)}{\sum_{(i_p^a, i_q^b) \in \mathcal{PI}_{ab} \cup \mathcal{NI}_{ab}} |w_{pq}|}, \quad (5)$$

where  $\alpha$  ( $0 \leq \alpha \leq 1$ ) is a damping factor controlling the degree that the reinforced user similarity relies on its previous value or the contributions of item similarities. At the beginning, the user pair  $(u_a, u_b)$  may or may not have direct PCC similarity. If not,  $sim(u_a, u_b)$  should only depend on the contributions of item similarities at the first step, i.e.,  $\alpha = 1$ . After that,  $\alpha$  should be set to its original value, a damping factor which is bigger than 0 and smaller than 1. Additionally, the parameter  $w_{pq}$  ( $p \in [P]$ ,  $q \in [Q]$ ) weighs the contribution of the item pair  $(i_p^a, i_q^b)$ .  $w_{pq}$  is regarded as the similarity between two ratings and defined as a linear function of the absolute rating difference  $d(r_{ai_p^a}, r_{bi_q^b})$ :

$$w_{pq} = 1 - 2d(r_{ai_p^a}, r_{bi_q^b}) = 1 - 2|r_{ai_p^a} - r_{bi_q^b}|, \quad (6)$$

where  $r_{ai_p^a}$  is the rating value given by user  $u_a$  to item  $i_p^a$ , and  $r_{bi_q^b}$  is the rating value given by user  $u_b$  to item  $i_q^b$ . All ratings should be first normalized by the following equation:

$$r_{nor} = \frac{r - r_{min}}{r_{max} - r_{min}}, \quad (7)$$

where  $r_{max}$  and  $r_{min}$  are the maximum and minimum rating values before normalization. After normalization, the absolute rating difference  $|r_{ai_p^a} - r_{bi_q^b}|$  falls within the interval  $[0, 1]$ . We split this interval  $[0, 1]$  into two subintervals,  $[0, 0.5]$  and  $(0.5, 1]$ . If  $|r_{ai_p^a} - r_{bi_q^b}|$  ranges from 0 to 0.5, the two ratings  $r_{ai_p^a}$  and  $r_{bi_q^b}$  are considered similar ( $0 \leq w_{pq} \leq 1$ ), with a smaller value of  $|r_{ai_p^a} - r_{bi_q^b}|$  indicating a larger value of  $w_{pq}$ . Given  $w_{pq} \geq 0$ , if the item pair  $(i_p^a, i_q^b)$  comes from  $\mathcal{PI}_{ab}$ , we derive  $w_{pq} \cdot sim(i_p^a, i_q^b) \geq 0$ . It means that similar ratings given to similar items contribute positively to the user similarity reinforcement. Otherwise, if  $(i_p^a, i_q^b)$  comes from  $\mathcal{NI}_{ab}$ , we derive  $w_{pq} \cdot sim(i_p^a, i_q^b) \leq 0$ , which means that similar ratings given to dissimilar items contribute negatively to the user similarity reinforcement. On the other hand, if  $|r_{ai_p^a} - r_{bi_q^b}|$  ranges from 0.5 to 1, the two ratings are regarded as dissimilar ( $-1 \leq w_{pq} < 0$ ). In this case, if the item pair  $(i_p^a, i_q^b)$  comes from  $\mathcal{PI}_{ab}$ , we derive  $w_{pq} \cdot sim(i_p^a, i_q^b) < 0$ . It means that dissimilar ratings given to similar items also contribute negatively to the user similarity reinforcement. Otherwise, if  $(i_p^a, i_q^b)$  comes from  $\mathcal{NI}_{ab}$ , we derive  $w_{pq} \cdot sim(i_p^a, i_q^b) > 0$ , which indicates that dissimilar ratings given to dissimilar items also contribute positively to the user similarity reinforcement.

Likewise, for two items  $i_k$  and  $i_l$ , the users who rated  $i_k$  and  $i_l$  are denoted by the two sets  $\mathcal{U}_k = \{u_1^k, u_2^k, \dots, u_C^k\}$  ( $\mathcal{U}_k \subset \mathcal{U}$ ) and  $\mathcal{U}_l = \{u_1^l, u_2^l, \dots, u_D^l\}$  ( $\mathcal{U}_l \subset \mathcal{U}$ ), respectively. Taking into account the contributions of users with positive or negative similarities, we define two sets,  $\mathcal{PU}_{kl} = \{(u_c^k, u_d^l) \mid (u_c^k, u_d^l) \in \mathcal{U}_k \times \mathcal{U}_l, sim(u_c^k, u_d^l) > 0\}$ , which includes all

user pairs from  $\mathcal{U}_k \times \mathcal{U}_l$  with positive similarities, and  $\mathcal{N}\mathcal{U}_{kl} = \{(u_c^k, u_d^l) \mid (u_c^k, u_d^l) \in \mathcal{U}_k \times \mathcal{U}_l, sim(u_c^k, u_d^l) < 0\}$ , which consists of user pairs from  $\mathcal{U}_k \times \mathcal{U}_l$  with negative similarities. Consequently, the similarity between  $i_k$  and  $i_l$  can be reinforced as:

$$sim(i_k, i_l) = (1 - \alpha) \cdot sim(i_k, i_l) + \alpha \cdot \frac{\sum_{(u_c^k, u_d^l) \in \mathcal{P}\mathcal{U}_{kl} \cup \mathcal{N}\mathcal{U}_{kl}} w_{cd} \cdot sim(u_c^k, u_d^l)}{\sum_{(u_c^k, u_d^l) \in \mathcal{P}\mathcal{U}_{kl} \cup \mathcal{N}\mathcal{U}_{kl}} |w_{cd}|}, \quad (8)$$

where  $w_{cd}$  ( $c \in [C]$ ,  $d \in [D]$ ) is the weight related to the user pair  $(u_c^k, u_d^l)$ , which is used to weigh the contribution of the similarity between  $u_c^k$  and  $u_d^l$ . Formally,  $w_{cd}$  is defined by:

$$w_{cd} = 1 - 2d(r_{u_c^k k}, r_{u_d^l l}) = 1 - 2|r_{u_c^k k} - r_{u_d^l l}|, \quad (9)$$

where  $r_{u_c^k k}$  is the normalized rating value given by user  $u_c^k$  to item  $i_k$ , and  $r_{u_d^l l}$  is that given by user  $u_d^l$  to item  $i_l$ .

**4.2.3. Mutually Enhancing Similarity Reinforcement Algorithm for Both Users and Items.** The similarity reinforcements for users and items are not isolated from each other. They can enhance each other. As shown in Equations (5) and (8), the user similarity reinforcement depends on item similarities, and vice versa. Therefore, we designed a comprehensive algorithm to incorporate the two processes into one and let them enhance each other. The entire procedure of the *Comprehensive Similarity Reinforcement* algorithm, or “CSR” for short, is summarized in Algorithm 1. In this algorithm,  $\mathbf{U}_{re\_sim}$  and  $\mathbf{I}_{re\_sim}$  denote the reinforced user and item similarity matrices.  $\mathbf{U}_{re\_sim\_last}$  and  $\mathbf{I}_{re\_sim\_last}$  record the values of  $\mathbf{U}_{re\_sim}$  and  $\mathbf{I}_{re\_sim}$  from the last iteration step. Line 1 calculates the traditional PCC-based user and item similarities. Lines 2 and 3 perform initialization. The iteration procedure is shown in lines 4 through 24. Within the iteration, lines 5 and 6 record the reinforced similarity matrices from the last step, and the user similarity reinforcement is performed by lines 7 through 12, and the item similarity reinforcement by lines 13 through 18. The iteration ends when the maximal number of iterations is reached (line 4) or the variations of the user similarity matrix and the item similarity matrix drop below a predefined threshold  $\epsilon$  (lines 19 through 23). Since similarity matrices are symmetric, we only need to perform one computation for each pair of symmetric elements in a similarity matrix, as shown in lines 10 and 16. The last line returns the reinforced user and item similarity matrices. In this algorithm, we store the similarity matrices on secondary storage. While performing similarity reinforcement for every pair of users and every pair of items, only the data involved in the present computation are loaded into memory. The computational complexity of the algorithm will be discussed later in Section 6.

### 4.3. Rating Prediction

In this section, we describe how we use the historical ratings from similar users and items, namely users and items with positive similarities, to make rating predictions. Suppose that  $u_a$  is an active user and  $i_k$  is a candidate item.  $u_b$  and  $i_l$  are the similar neighbors of  $u_a$  and  $i_k$ , respectively. The user-based and item-based rating prediction results for the active user  $u_a$  on the candidate item  $i_k$  are presented by the following two equations:

$$\hat{r}_{ak}^u = \bar{r}_a + \frac{\sum_{u_b \in \mathcal{N}_{ua}} \mathbf{U}_{re\_sim}(a, b) \cdot (r_{bk} - \bar{r}_b)}{\sum_{u_b \in \mathcal{N}_{ua}} \mathbf{U}_{re\_sim}(a, b)}, \quad (10)$$

**ALGORITHM 1:** CSR Algorithm

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**Input:** user-item rating matrix  $\mathbf{R}$ , damping factor  $\alpha$ , the maximum number of iteration  $iter\_max$ , threshold  $\epsilon$ .

**Output:** reinforced user similarity matrix  $\mathbf{U}_{re\_sim}$ , reinforced item similarity matrix  $\mathbf{I}_{re\_sim}$ .

```

1  compute user similarity matrix  $\mathbf{U}_{sim}$  and item similarity matrix  $\mathbf{I}_{sim}$  based on PCC;
2   $\mathbf{U}_{re\_sim} \leftarrow \mathbf{U}_{sim}$ ;
3   $\mathbf{I}_{re\_sim} \leftarrow \mathbf{I}_{sim}$ ;
4  for  $iter\_cur \leftarrow 1$  to  $iter\_max$  do
5     $\mathbf{U}_{re\_sim, last} \leftarrow \mathbf{U}_{re\_sim}$ ;
6     $\mathbf{I}_{re\_sim, last} \leftarrow \mathbf{I}_{re\_sim}$ ;
7    for  $a \leftarrow 1$  to  $M$  do
8      for  $b \leftarrow a + 1$  to  $M$  do
9        Update  $\mathbf{U}_{re\_sim}(a, b)$  according to Equation (5);
10        $\mathbf{U}_{re\_sim}(b, a) \leftarrow \mathbf{U}_{re\_sim}(a, b)$ ;
11     end
12   end
13   for  $k \leftarrow 1$  to  $N$  do
14     for  $l \leftarrow k + 1$  to  $N$  do
15       Update  $\mathbf{I}_{re\_sim}(k, l)$  according to Equation (8);
16        $\mathbf{I}_{re\_sim}(l, k) \leftarrow \mathbf{I}_{re\_sim}(k, l)$ ;
17     end
18   end
19    $d_U = \|\mathbf{U}_{re\_sim} - \mathbf{U}_{re\_sim, last}\|_F$ ;
20    $d_I = \|\mathbf{I}_{re\_sim} - \mathbf{I}_{re\_sim, last}\|_F$ ;
21   if  $d_U < \epsilon$  and  $d_I < \epsilon$  then
22     break;
23   end
24 end
25 return  $\mathbf{U}_{re\_sim}$  and  $\mathbf{I}_{re\_sim}$ ;

```

---

and

$$\hat{r}_{ak}^i = \bar{r}_k + \frac{\sum_{i_l \in \mathcal{N}_{i_k}} \mathbf{I}_{re\_sim}(k, l) \cdot (r_{al} - \bar{r}_l)}{\sum_{i_l \in \mathcal{N}_{i_k}} \mathbf{I}_{re\_sim}(k, l)}, \quad (11)$$

where  $\mathcal{N}_{u_a} = \{u_b \mid \mathbf{U}_{re\_sim}(a, b) > 0, a \neq b\}$  and  $\mathcal{N}_{i_k} = \{i_l \mid \mathbf{I}_{re\_sim}(k, l) > 0, k \neq l\}$  are the sets of similar neighbors of  $u_a$  and  $i_k$ , respectively.

In order to merge the user-based and item-based predictions into a synthesized result, we employ two confidence weights  $con_u$  and  $con_i$  [56, 57] to balance the two prediction results. The confidence weights are defined by:

$$con_u = \sum_{u_b \in \mathcal{N}_{u_a}} \frac{\mathbf{U}_{re\_sim}(a, b)}{\sum_{u_b \in \mathcal{N}_{u_a}} \mathbf{U}_{re\_sim}(a, b)} \cdot \mathbf{U}_{re\_sim}(a, b), \quad (12)$$

and

$$con_i = \sum_{i_l \in \mathcal{N}_{i_k}} \frac{\mathbf{I}_{re\_sim}(k, l)}{\sum_{i_l \in \mathcal{N}_{i_k}} \mathbf{I}_{re\_sim}(k, l)} \cdot \mathbf{I}_{re\_sim}(k, l). \quad (13)$$

Finally, the missing rating value is predicted by:

$$\hat{r}_{ak} = h_u \cdot \hat{r}_{ak}^u + h_i \cdot \hat{r}_{ak}^i, \quad (14)$$

where  $h_u$  and  $h_i$  [56, 57] are the weights of user-based and item-based prediction results and defined by:

$$h_u = \frac{\lambda \cdot con_u}{\lambda \cdot con_u + (1 - \lambda) \cdot con_i}, \quad (15)$$

and

$$h_i = \frac{(1 - \lambda) \cdot con_i}{\lambda \cdot con_u + (1 - \lambda) \cdot con_i}, \quad (16)$$

where  $\lambda$  ( $0 \leq \lambda \leq 1$ ) is a parameter employed to determine how much the final prediction result depends on the user-based and item-based prediction results.

## 5. EVALUATION

In this section, we conduct comprehensive experiments on three publicly available datasets to evaluate the effectiveness of the proposed approach to alleviate data sparsity.

### 5.1. Dataset

The three publicly available datasets used in our experiments are MovieLens 1M, Netflix and Yahoo! Music. All the three datasets contain ratings in a 1 to 5 scale. MovieLens 1M contains 1,000,209 ratings by 6,040 users of total 3,900 movies. Netflix contains more than 100 million ratings by 480,189 customers of 17,770 movies. Yahoo! Music contains 311,704 ratings by 15,400 users of 1,000 songs. The sparsity levels of the three datasets are 95.8%, 98.8%, and 98%, respectively.

### 5.2. Evaluation Metric

We adopt two widely used metrics, Mean Absolute Error (MAE) and Root Mean Squared Error (RMSE), to measure the prediction accuracy of CF algorithms. The two metrics are particularly useful in the context of evaluating prediction accuracy in off-line tests. The first measure (MAE) treats every error equally, while the second one (RMSE) emphasizes larger errors. Let  $\mathcal{T}$  denote the set of ratings to be predicted, i.e.,  $\mathcal{T} = \{(p, q) \mid p \in [M], q \in [N], r_{pq} \text{ to be predicted}\}$ . Then, the MAE and RMSE metrics are defined by:

$$MAE = \frac{1}{|\mathcal{T}|} \cdot \sum_{(p,q) \in \mathcal{T}} |\hat{r}_{pq} - r_{pq}|, \quad (17)$$

and

$$RMSE = \sqrt{\frac{1}{|\mathcal{T}|} \cdot \sum_{(p,q) \in \mathcal{T}} (\hat{r}_{pq} - r_{pq})^2}, \quad (18)$$

where  $r_{pq}$  and  $\hat{r}_{pq}$  are the actual and predicted rating values, respectively. A smaller value of MAE or RMSE indicates higher prediction accuracy.

### 5.3. Performance Comparison

In the experiments, we studied the prediction accuracy of our approach under different data sparsity levels and compared our approach with five well-known CF methods. They are the User-based CF using PCC (UPCC) [3], the Item-based CF using PCC (IPCC) [38], the hybrid CF using PCC [57], the Regularized Matrix Factorization (RMF) [25] and the Regularized Matrix Factorization with Biases (BRMF) [41]. These methods have been widely used as baseline methods in the study of recommender systems [39, 42, 49].

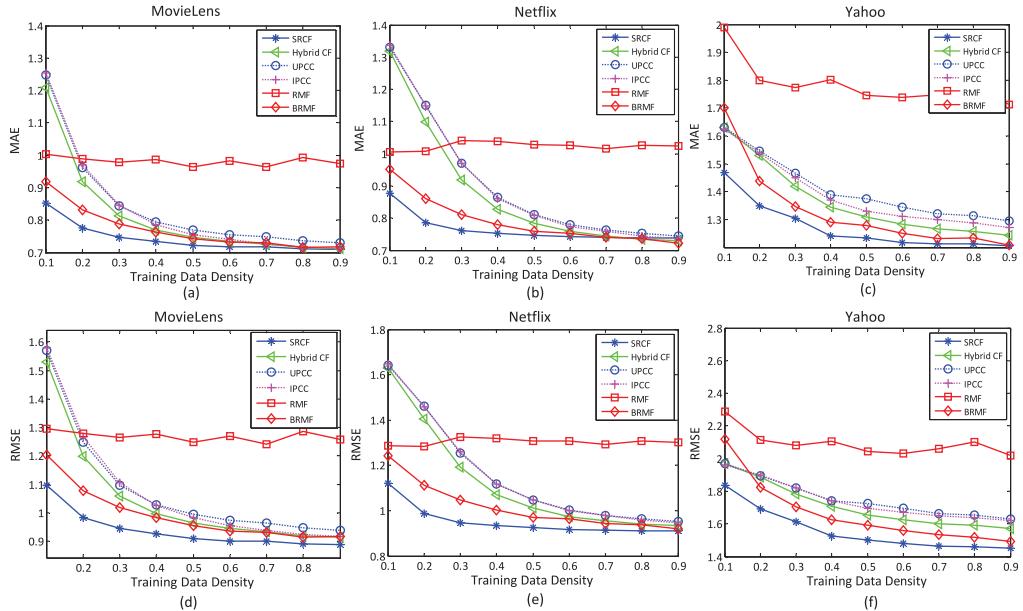


Fig. 4. MAE and RMSE comparison between our approach and traditional CF algorithms.

In the three experimental datasets, users and items without any rating information were filtered out. After that, about 70% of the data of each dataset were selected as training data, and the remaining 30% were used as test data. Given a training set, in order to simulate different sparsity levels, we randomly removed some of its entries to make the training data density vary from 10% to 90%, with a step value of 10%. Finally, we applied different methods to the prepared training sets, and evaluated the prediction accuracy of different methods on the corresponding test datasets. The parameter  $\alpha$  is set to 0.5, 0.5, and 0.3 and  $\lambda$  is set to 0.6, 0.5, and 0.7 for the three datasets, respectively, because these values yield the optimal performance. We will investigate the impacts of the two parameters in Sections 5.4 and 5.5.

The comparison of prediction accuracy between different methods is illustrated in Figure 4. Figures 4(a) to (c) report the MAE comparison of different methods on MovieLens 1M, Netflix, and Yahoo! Music datasets, respectively. Figures 4(d) to (f) show the RMSE comparison of different methods on the three datasets. We can see that the proposed similarity reinforcement technique improves the rating prediction accuracy significantly compared with the traditional CF algorithms, especially when the training data become sparser. That testifies the excellent capability of our approach to handle data sparsity.

The boxplots in Figure 5 show another observation that the variation of prediction errors yielded by the proposed approach is significantly lower than those by other methods. For example, on the MovieLens 1M dataset, 50% of the absolute prediction errors are within (0.31, 1.12) with a range = 0.81 for the proposed approach, within (0.36, 1.43) with a range = 1.07 for RMF, within (0.32, 1.18) with a range = 0.86 for BRMF, within (0.36, 1.40) with a range = 1.04 for the hybrid CF, within (0.37, 1.48) with a range = 1.11 for UPCC, and within (0.37, 1.49) with a range = 1.12 for IPCC. It is similar on the other two datasets. The results indicate that the prediction accuracy of the proposed approach is more stable than that of traditional CF algorithms across different datasets.

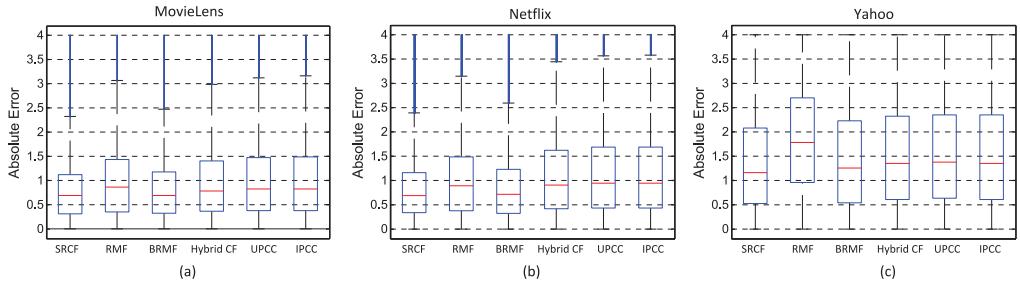


Fig. 5. Boxplots of absolute prediction errors of different methods.

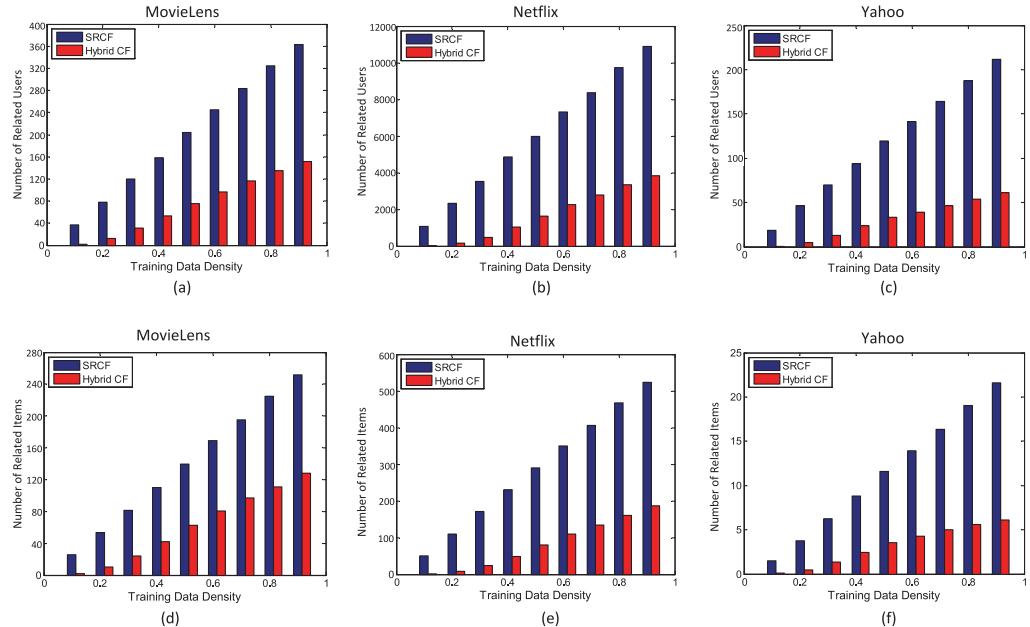
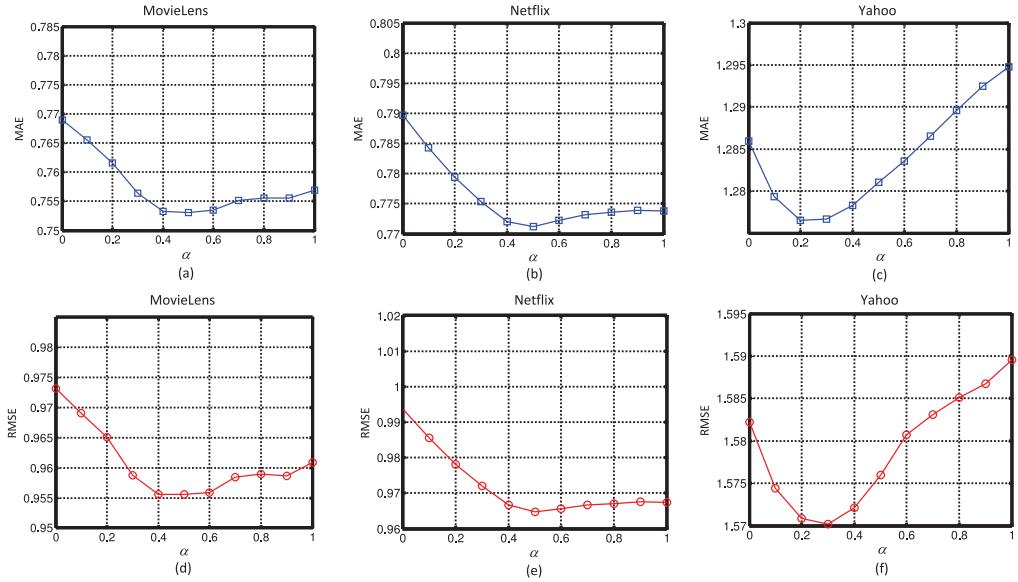


Fig. 6. Average number of related users and items.

Furthermore, we recorded the average numbers of related users and related items discovered by different approaches under different training data densities and report the results in Figure 6 (Given an active user and a candidate item, the similar neighbors of the active user who also rated the candidate item are called *related users*. The similar neighbors of the candidate item that were also rated by the active user are called *related items*.) We can see that our approach can discover more related users and items than the hybrid CF can. Since UPCC and IPCC detected the same number of related users and related items, respectively, as the hybrid CF did, we do not present the results of the two methods. In conclusion, our approach can find more related users and items using the comprehensive similarity reinforcement mechanism, meaning that our approach can extract more historical information from related neighbors, which helps to achieve higher prediction accuracy.

#### 5.4. Impact of $\alpha$

The damping factor  $\alpha$  is used to control the degree that the reinforced similarity relies on its previous value or the contribution of the reinforcement procedure. In this section,

Fig. 7. Impact of  $\alpha$ .

in order to investigate the impact of different values of  $\alpha$ , we set another adjustable parameter  $\lambda = 0.5$  and varied  $\alpha$  from 0 to 1 with a step value of 0.1 and investigated how MAE and RMSE change with  $\alpha$  under three different training data densities, 0.3, 0.6, and 0.9. Figures 7(a) to (c) illustrate the relationship between the average MAE under three densities and  $\alpha$ . Figures 7(d) to (f) show the relationship between the average RMSE and  $\alpha$ . We can see from Figure 7 that the  $\alpha$  value has impacts on the prediction accuracy. The optimal values of  $\alpha$  on the three datasets are 0.5, 0.5, and 0.3.

### 5.5. Impact of $\lambda$

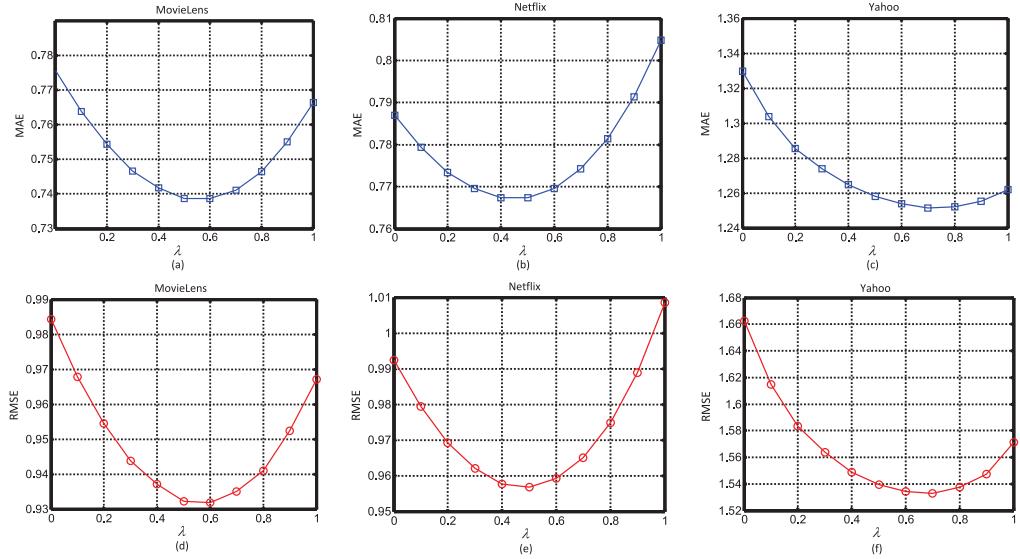
The parameter  $\lambda$  provides our prediction with the flexibility to vary the dependence on the user-based and item-based prediction results. If  $\lambda = 1$ , we only consider information from similar users; if  $\lambda = 0$ , we only consider historical information from similar items; if  $0 < \lambda < 1$ , we fuse information from both similar users and similar items to predict missing ratings for active users.

To study the impact of the parameter  $\lambda$  on the performance of our CF method, we set another adjustable parameter  $\alpha$  equal to 0.5, 0.5, and 0.3 on the three datasets, respectively, and varied  $\lambda$  from 0 to 1 with a step value of 0.1 and investigated the prediction accuracy of our algorithm under three different training data densities, 0.3, 0.6, and 0.9. Figures 8(a) to (c) show the relationship between the average MAE under three densities and  $\lambda$ , and Figures 8(d) to (f) show the relationship between the average RMSE and  $\lambda$ .

In Figure 8, we observe that the value of  $\lambda$  impacts the recommendation results, which calls for a suitable  $\lambda$  value for higher prediction accuracy. The optimal  $\lambda$  values are 0.6, 0.5, and 0.7 on the three datasets, respectively.

## 6. COMPUTATIONAL COMPLEXITY DISCUSSION

In this section, we compare the computational complexity of our approach with that of MF approaches.

Fig. 8. Impact of  $\lambda$ .

**Computational Complexity of SRCF.** There are three main procedures in our SRCF approach, the PCC similarity computing, the similarity reinforcement, and the rating prediction. We analyze the computational complexities of the three procedures:

**PCC similarity computing.** Suppose that  $|\bar{I}_u|$  ( $|\bar{I}_u| \ll N$ ) is the average number of items that each user has rated and  $|\bar{U}_i|$  ( $|\bar{U}_i| \ll M$ ) is the average number of users that each item has interacted with. The time complexity of user similarity computation based on Equations (1) and (3) is  $O(|\bar{I}_u|)$ , since there are at most  $|\bar{I}_u|$  co-rated items between two users. Similarly, the time complexity of item similarity computation based on Equations (2) and (4) is  $O(|\bar{U}_i|)$ , since there are at most  $O(|\bar{U}_i|)$  common users between two items. Based on that, the time complexity of the similarity computation for a total of  $M^2$  pairs of users is  $O(M^2|\bar{I}_u|)$ . Similarly, the time complexity of the similarity computation for a total of  $N^2$  pairs of items is  $O(N^2|\bar{U}_i|)$ .

**Similarity reinforcement.** After similarity computing, user and item similarity reinforcements are performed using Equations (5) and (8). When performing similarity reinforcement between each pair of users, the similarities between a total of  $|\bar{I}_u|^2$  pairs of items should be examined. Suppose that the maximum number of iterations is  $T$ . The time complexity for user similarity reinforcement is  $O(TM^2|\bar{I}_u|^2)$ . Similarly, the time complexity for item similarity reinforcement is  $O(TN^2|\bar{U}_i|^2)$ .

**Rating prediction.** The time complexity of the user-based prediction for a missing rating value is  $O(M)$ , since at most  $M$  similar users are used for prediction. Similarly, the time complexity of the item-based prediction for a missing rating value is  $O(N)$ , since at most  $N$  similar items are used for prediction. The hybrid prediction is a linear combination of the user-based and item-based predictions, so its time complexity is  $O(M + N)$ .

**Computational Complexity of MF.** In comparison, MF approaches map both users and items to a joint latent factor space of dimensionality  $f$  ( $f \ll M, N$ ), and user-item interactions are modeled as inner products in that space. Accordingly, each user  $u$  is associated with a vector  $\mathbf{p}_u \in \mathbb{R}^f$  and each item  $i$  is associated with a vector  $\mathbf{q}_i \in \mathbb{R}^f$ . MF approaches mainly include two procedures: model training and rating prediction.

**Model training.** In the model training procedure, *stochastic gradient descent* is a popular method. Given a training case (*userId*, *itemId*, *rating*), the algorithm predicts the rating value, computes the prediction error, and updates the corresponding user vector and item vector in the opposite direction of the gradient. Suppose that the maximum number of iteration is  $T_1$ . The time complexity of the model training procedure is  $O(2T_1Kf)$ , where  $K$  is the number of training cases, which is approximately equal to  $M|\bar{I}_u|$  or  $N|\bar{U}_i|$ .

**Rating prediction.** The rating prediction procedure uses the dot product of a user vector and an item vector to denote the user's overall interest in this item. Therefore, the time complexity of predicting a rating value for an active user on a candidate item is  $O(f)$ , where  $f$  is the dimensionality of the latent factor space.

Based on the above analysis, the time complexity of the similarity computation and reinforcement of our SRCF approach is  $O(M^2|\bar{I}_u| + N^2|\bar{U}_i| + TM^2|\bar{I}_u|^2 + TN^2|\bar{U}_i|^2)$ . Because  $f$  is much smaller than  $M$  and  $N$ , and  $K$  is approximately equal to  $M|\bar{I}_u|$  or  $N|\bar{U}_i|$ , the time complexity of the similarity computing and reinforcement of SRCF is higher than the time complexity ( $O(2T_1Kf)$ ) of the model training of MF approaches, even though the iteration number  $T$  in our approach is usually smaller than  $T_1$  in MF approaches. Additionally, the time complexity of the rating prediction of our approach is  $O(M + N)$ , which is also higher than that of MF approaches ( $O(f)$ ). Moreover, our algorithm needs to store an  $M \times M$  user similarity matrix and an  $N \times N$  item similarity matrix, while MF approaches only need to store  $M f$ -dimensional user vectors and  $N f$ -dimensional item vectors ( $f \ll M, N$ ), so the storage requirement of our approach is higher than that of MF. Both the execution time and storage requirements of the two kinds of approaches are determined by their algorithm mechanisms. In the future, we will optimize our algorithm as well as explore the possibility to accelerate it by means of parallel computing.

## 7. CONCLUSIONS

In this article, we proposed a novel similarity reinforcement mechanism to augment traditional memory-based CF approaches for tackling the data sparsity problem. The main advantage of our approach is that it does not rely on any auxiliary data source. It makes better use of known but limited user-item rating information to reveal more implicit similarity relationships between users or items. The more similar neighbors discovered by our algorithm, the more abundant historical data are provided, and the more reliable and accurate missing rating predictions can be made. Additionally, our approach provides an integrated framework for the user and item similarity reinforcements that allows them to enhance each other. Experimental results show that our approach can achieve higher prediction accuracy than traditional memory-based and model-based CF methods. However, the work presented in this article still has some limitations. Our approach cannot handle cold-start users or items that do not have any rating information. Meanwhile, how to guarantee good performance in terms of execution time and storage when applying our algorithm to very large datasets is still a problem. In the future, we will optimize the algorithm as well as explore the possibility to accelerate it by means of parallel computing.

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